Annuity Due

The present value of an n period annuity due is denoted by $\ddot{a}_{\overline{n}|}$

$$\begin{aligned} PV &= \ddot{a}_{\overline{n}|} \\ &= 1 + v^2 + \cdots + v^{n-1} \\ &= \frac{1-v^n}{1-v} \\ &= \frac{1-v^n}{d} \end{aligned}$$

The present value of an n period annuity due is,

$$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$$

Exercise: Write R function code that computes the present value of annuity due.

```
pv_due_ann = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a * r^(i-1)
return(x)
}
pv_due_ann(1,0.05,10)
```

```
## [1] 8.107822
```

Exercise: Write R function code that computes the future value of annuity due.

```
fv_due_ann = function(a, i, n) {
x = 0
r=(1+i)
for(i in 1:n) x = x + a * r^(i)
return(x)
}
fv_due_ann(1,0.06,15)
```

```
## [1] 24.67253
```

Increasing Annuities

An annuity whose n payments are 1,2,. . . ,n is called a *unit increasing immediate annuity.* If paymnets are made at the end of each period, the annuity is immediate and is denoted by $(Ia)_{\overline{n}|}$

$$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \cdots + nv^n$$

$$\begin{aligned} PV &= (Ia)_{\overline{n}|} \\ &= \frac{\ddot{a}_{\overline{n}|} - nv^n}{i} \end{aligned}$$

Exercise: Write R function code that computes the present value of increasing annuity immediate.

```
pv_imm_inc = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a*i * r^(i)
return(x)
}
pv_imm_inc(1,0.05,4)
```

```
## [1] 8.648763
```

```
pv_imm_inc(1,0.06,15)
```

```
## [1] 67.2668
```

If payments are made at the beginning of each period, the annuity is due and is denoted by $(I\ddot{a})_{\overline{n}|}$

$$(I\ddot{a})_{\overline{n}|} = 1 + 2v^2 + 3v^3 + \cdots + nv^{n-1}$$

Exercise: Write R function code that computes the present value of increasing annuity due

```
pv_due_inc = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a*i * r^(i-1 )
return(x)
}
pv_due_inc(1,0.05,4)
```

```
## [1] 9.081201
```

```
pv_due_inc(1,0.06,15)
```

```
## [1] 71.30281
```

The future value of an increasing unit annuity immediate is denoted by $(Is)_{\overline{n}|}$

$$(Is)_{\overline{n}|} = (1+i)^{n-1} + 2(1+i)^{n-2} + 3(1+i)^{n-3} + \cdots + (n-1)(1+i) + n$$

The future value of an increasing unit annuity due is denoted by $(I\ddot{s})_{\overline{n}|}$

$$(I\ddot{s})_{\overline{n}|} = (1+i)^{n} + 2(1+i)^{n-1} + 3(1+i)^{n-2} + \cdots + (n-1)(1+i)^2 + n(1+i)$$

Exercise: Write R function codes that computes the future value of increasing annuity immediate and due.

```
fv_imm_inc = function(a, i, n) {
x = 0
r=(1+i)
for(i in 1:n) x = x + a*i* r^(n-i)
return(x)
}
fv_imm_inc(1,0.06,15)
```

```
## [1] 161.2088
```

```
fv_due_inc = function(a, i, n) {
x = 0
r=(1+i)
for(i in 1:n) x = x + a *i* r^(n-i+1)
return(x)
}
fv_due_inc(1,0.06,15)
```

```
## [1] 170.8813
```

Decreasing Annuities

The unit decreasing immedaite annuity has n payments: n, *n-1*, ... , 1. Its present value is denoted by $(Ds)_{\overline{n|}}$

$$(Ds)_{\overline{n|}} = nv + (n-1)v^2 + (n-2)v^3 + \cdots + 2v^{n-1} + v^n$$

The unit decreasing due annuity has n payments: n, *n-1*, ... , 1. Its present value is denoted by $(D\ddot{s})_{\overline{n|}}$

$$(D\ddot{s})_{\overline{n|}} = n + (n-1)v + (n-2)v^2 + \cdots + 2v^{n-2} + v^{n-1}$$

Exercise: Write R function code that computes the present value of decreasing annuity immediate.

```
pv_imm_dec = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a*(n-i+1) * r^(i)
return(x)
}

pv_imm_dec(1,0.06,15)
```

```
## [1] 88.12918
```

General Formula for Annuities with Terms in Arithmetic Progression

Suppose the first payment in an annuity immediate is P>0 and the subsequent payments change by Q per period, where Q can be either positive or negative. If the annuity has n payments, the sequence of payments is $P, P+Q, P+2Q, \ldots, P+(n-1)Q$. It can be shown that the present value of this annuity is

$$Pa_{\overline{n|}} + Q\left(\frac{a_{\overline{n|}} - nv^n}{i}\right)$$

```
pv_imm_prog<-function(P,Q,i,n){
  x=0
  y=0
  r=1/(1+i)
  for(i in 1:n) x=x+P*r^i
```

```
  for(t in 2:n) y=y+Q*(t-1)*r^t
  return(x+y)
}
pv_imm_prog(100,200,0.06,15)
```

```
## [1] 12482.14
```